

Example 7d: Woven Composite Analysis – Single Step

This example problem and the next illustrate MAC/GMC 4.0's ability to analyze composites with woven (or braided) reinforcements. The present example problem performs the analysis using a one step homogenization procedure. The next example problem uses a two step procedure that gives improved results. Triply periodic GMC has the ability to represent the inherently three-dimensional architecture of the reinforcement in woven composites. Currently, woven composites may be treated within MAC/GMC 4.0 by employing the internal constitutive models that allow specification of an arbitrary direction of transverse isotropy ($CMOD = 3, 7, \text{ or } 9$). Then, by choosing the direction vector appropriately for the material occupying each subcell, the woven reinforcement architecture can be represented.

Woven polymer matrix composites (PMCs), such as graphite/epoxy, are a common type of woven composite. The polymer matrix in these composites is often treated as linearly elastic. An important class of woven PMC is considered in this example problem: plain weave reinforced PMCs. The plain weave architecture involves fibers, or more commonly tows of fibers, in a repeating over-one, under-one pattern as shown in Figure 7.7. As indicated in the figure, an RUC can be identified from the architecture that remains unchanged when the plain weave reinforcement is infiltrated with a polymer matrix to form a composite. The triply periodic GMC RUC representation of the plain weave composite is shown in Figure 7.8, with an exploded view given in Figure 7.9. As shown, the subcells within the triply periodic RUC are themselves occupied by composite materials that represent the fiber tows of the weave. In addition, some subcells are occupied by the pure matrix material. Figure 7.9 clearly shows the three-dimensional nature of the woven reinforcement as the fiber tows undulate in and out of each other.

In this example problem, the composite material within subcells is represented using the transversely isotropic elastic model with arbitrary plane of isotropy ($CMOD = 9$). The ability to employ the arbitrary direction of transverse isotropy allows the representation of the subcells containing inclined fibers in Figure 7.9. The material system considered is graphite/epoxy. The effective properties of the composite subcells have been taken from Example 1a (see Table 1.1), which determined the effective properties of a 0.65 fiber volume fraction unidirectional graphite/epoxy composite. Thus, in the present example problem, it is assumed that the infiltrated fiber tows that occupy the subcells have a fiber volume fraction of 0.65. Due to the chosen dimension (a , g , and H in Figure 7.8) this results in an overall woven composite fiber volume fraction of 0.325.

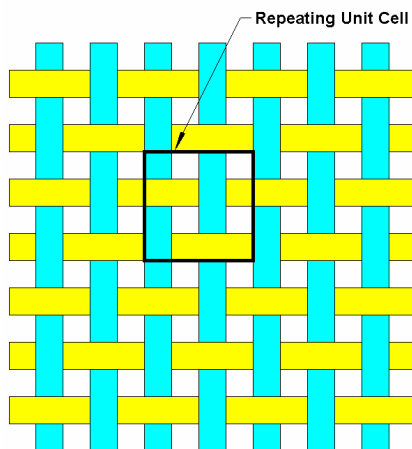


Figure 7.7 Top view of the plain weave architecture with the repeating unit cell identified.

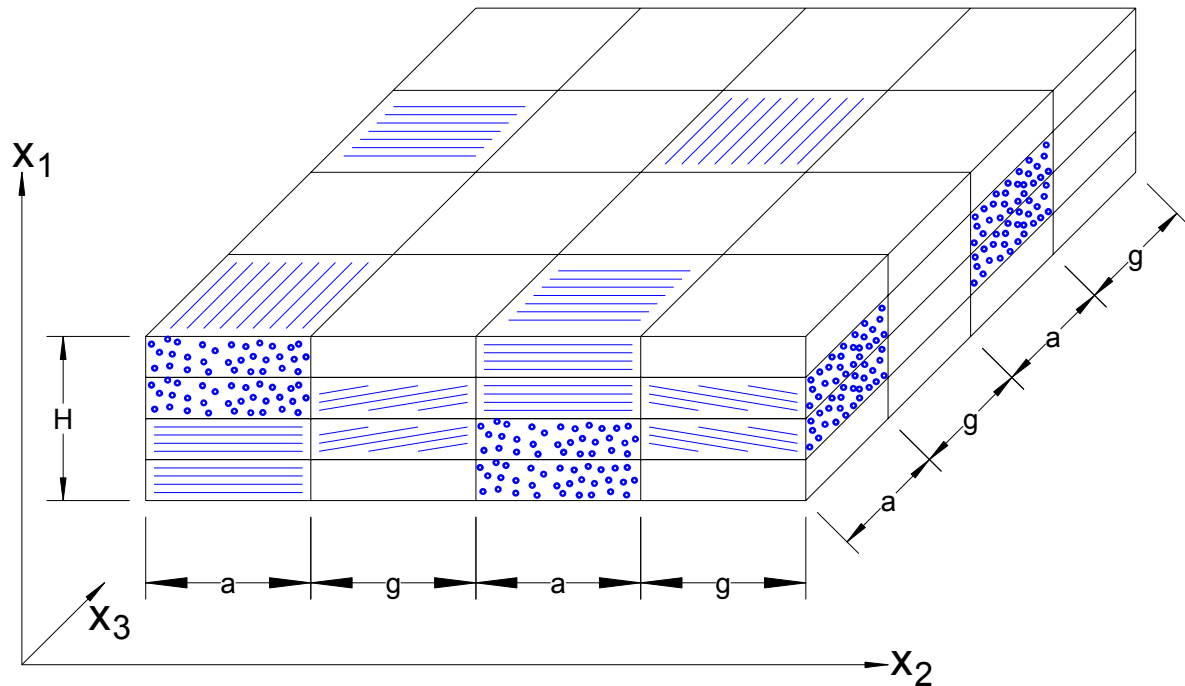


Figure 7.8 MAC/GMC 4.0 triply periodic RUC that represents a plain weave reinforced composite.

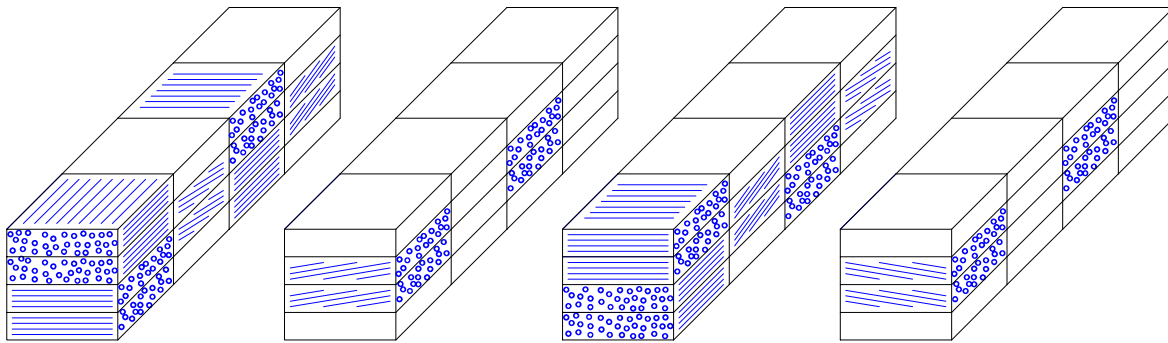


Figure 7.9 MAC/GMC 4.0 triply periodic RUC that represents a plain weave reinforced composite – exploded view.

MAC/GMC Input File: **example_7d.mac**

MAC/GMC 4.0 Example 7d - graphite/epoxy plain weave reinforced composite

***CONSTITUENTS**

NMATS=7

M=1 CMOD=9 MATID=U MATDB=1 &

EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=0.,0.,1.

M=2 CMOD=9 MATID=U MATDB=1 &

EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=0.,1.,0.

```

M=3 CMOD=9 MATID=U MATDB=1 &
  EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=0.25,1.,0.
M=4 CMOD=9 MATID=U MATDB=1 &
  EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=-0.25,1.,0.
M=5 CMOD=6 MATID=U MATDB=1 &
  EL=3.45E9,3.45E9,0.35,0.35,1.278E9,45.E-6,45.E-6
M=6 CMOD=9 MATID=U MATDB=1 &
  EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=0.25,0.,1.
M=7 CMOD=9 MATID=U MATDB=1 &
  EL=253.5E9,6.05E9,0.3901,0.4682,4.167E9,-0.4724E-6,26.63E-6 D=-0.25,0.,1.
*RUC
MOD=3 ARCHID=99
NA=4 NB=4 NG=4
D=0.25,0.25,0.25,0.25
H=1.,1.,1.,1.
L=1.,1.,1.,1.
# -- gamma = 1
SM=1,5,2,5
SM=1,3,2,4
SM=2,3,1,4
SM=2,5,1,5
# -- gamma = 2
SM=5,5,5,5
SM=7,5,6,5
SM=7,5,6,5
SM=5,5,5,5
# -- gamma = 3
SM=2,5,1,5
SM=2,4,1,3
SM=1,4,2,3
SM=1,5,2,5
# -- gamma = 4
SM=5,5,5,5
SM=6,5,7,5
SM=6,5,7,5
SM=5,5,5,5
*PRINT
NPL=-1
*END

```

Annotated Input Data

1) Flags: None

2) Constituent materials (***CONSTITUENTS**) [KM_2]:

Number of materials:	7	(NMATS=7)
Constitutive models:	Arbitrary transversely isotropic	(CMOD=9)
	Elastic	(CMOD=6)
Materials:	User-defined	(MATID=U)
Material property source:	Read from input file	(MATDB=1)
Material properties:	See Table 7.1	(EL=...)

Direction of trans. Isotropy: Material #1: (0, 0, 1) (D=0., 0., 1.)
 Material #2: (0, 1, 0) (D=0., 1., 0.)
 Material #3: (0.25, 1, 0) (D=0.25, 1., 0.)
 Material #4: (-0.25, 1, 0) (D=-0.25, 1., 0.)
 Material #6: (0.25, 0, 1) (D=0.25, 0., 1.)
 Material #7: (-0.25, 0, 1) (D=-0.25, 0., 1.)

Table 7.1 Constituent material properties for example 7d.

	E_A (GPa)	E_T (GPa)	ν_A	ν_T	G_A (GPa)	α_A ($10^{-6}/^{\circ}\text{C}$)	α_T ($10^{-6}/^{\circ}\text{C}$)
Composite	253.5	6.05	0.3901	0.4682	4.167	-0.4724	6.63
Epoxy	3.45	3.45	0.35	0.35	1.278	45.	45.

In this example problem, the material occupying many of the subcells represents a unidirectional composite material (see [Figure 7.8](#) and [Figure 7.9](#)) intended to model the infiltrated graphite/epoxy fiber tow of which the woven composite reinforcement is composed. Thus, transversely isotropic effective properties for the unidirectional composite material, taken from the results of Example 1a, are specified (for materials #1 – 4 and 6 – 7). Material #5 is the pure isotropic epoxy matrix material. In order to account for the directionality of the fibers in each subcell, the direction vector (D=) is used to specify the direction of transverse isotropy for each of the six transversely isotropic constituent materials. As indicated, a slope of $\frac{1}{4}$ (D=0.25,...) has been employed for the inclined fibers in the appropriate subcells. When placed in the correct arrangement (under ***RUC**), these constituent materials will represent the plain weave reinforced composite shown in [Figure 7.8](#). For more information on constitutive material properties, see the MAC/GMC 4.0 Keywords Manual Section 2.

3) Analysis type (***RUC**) → Repeating Unit Cell Analysis [KM_3]:

Analysis model: Triply periodic GMC (MOD=3)
 RUC architecture: User-defined (ARCHID=99)
 No. subcells in x1-dir.: 4 (NA=4)
 No. subcells in x2-dir.: 4 (NB=4)
 No. subcells in x3-dir.: 4 (NG=4)
 Subcell depths: 0.25, 0.25, 0.25, 0.25 (D=0.25, 0.25, 0.25, 0.25)
 Subcell heights: 1., 1., 1., 1. (H=1., 1., 1., 1.)
 Subcell lengths: 1., 1., 1., 1. (L=1., 1., 1., 1.)
 Material assignment: see input file (SM=...)

4) Loading: None

5) Damage and Failure: None

6) Output:

a) Output file print level (***PRINT**) [KM_6]:

Print level: -1 (effective properties only) (NPL=-1)

b) x-y plots (***XYPLOT**): None

7) End of file keyword: (***END**)

Results

The results from this example problem are the predicted effective properties of the woven composite printed to the MAC/GMC 4.0 output file:

CG - Effective/Macro Stiffness Matrix

NOTE: Stiffness relates stresses to normal and ENGINEERING SHEAR STRAINS

0.6650E+10	0.3432E+10	0.3432E+10	0.0000E+00	0.3260E-08	0.0000E+00
0.3432E+10	0.1061E+11	0.2873E+10	0.0000E+00	-0.1010E-26	-0.9413E-08
0.3432E+10	0.2873E+10	0.1061E+11	0.0000E+00	-0.4891E-09	-0.9458E-09
0.0000E+00	0.0000E+00	0.0000E+00	0.2079E+10	0.0000E+00	0.0000E+00
0.1281E-08	0.1164E-09	-0.3193E-09	0.0000E+00	0.1859E+10	0.0000E+00
-0.4657E-09	-0.1791E-09	-0.1979E-08	0.0000E+00	0.5404E-28	0.1859E+10

Effective Engineering Moduli

Use with caution when global stiffness matrix is anisotropic

E11S= 0.4902E+10
 N12S= 0.2545
 E22S= 0.8703E+10
 N23S= 0.1245
 E33S= 0.8703E+10
 G23S= 0.2079E+10
 G13S= 0.1859E+10
 G12S= 0.1859E+10

Effective Thermal Expansion Coefficients

NOTE: Shear CTEs are "engineering" shear CTEs

0.4267E-04	0.2045E-04	0.2045E-04
0.0000E+00	-0.2846E-22	0.3251E-22

The effective stiffness matrix results show that, even through materials #3, #4, #6, and #7 are monoclinic (in the global coordinates of the woven composite), the woven composite is orthotropic. This is because for every subcell that contains inclined fibers, there is a subcell containing fibers with the incline reversed (see [Figure 7.9](#)). The anisotropic terms thus add to zero during the GMC triply periodic homogenization procedure. The same is true of the “shear” CTE terms.

For a woven composite such as the plain weave graphite/epoxy modeled here, the in-plane properties (i.e., x_2 - x_3 plane) properties are the most important. This is because woven composites are often made in thin plate form. Thus, the in-plane properties for the woven composite predicted by this single step GMC homogenization procedure are:

$$E = 8.703 \text{ GPa}, \nu = 0.1245, G = 2.079 \text{ GPa}, \text{ and } \alpha = 20.45 \times 10^{-6} / ^\circ\text{C}$$

Thus, these predictions indicate that the in-plane elastic modulus of 8.703 GPa is quite small, on the order of the transverse elastic modulus of the fiber (7.6 GPa) and the elastic modulus of the epoxy (3.45 GPa). The extreme longitudinal stiffness of the fiber (388.2 GPa) appears not to impart much stiffness to the woven composite in this single step homogenization procedure. For an alternative two step procedure, which has been shown to be significantly more accurate (particularly in the case of large constituent property mismatch), see Example 7e.